1/2/18 "A mistake is food for a new invention." -Anonymous

HW: "Curve Sketching HW" #1, 2
Test 3 on Friday 1/12
Midterm on Thursday 1/18

AIM: How do we Sketch Curves?

Warm Up:

CURVE SKETCHING

Guidelines for Analyzing the Graph of a Function

1. Find domain and range.
2. Find *x*- and *y*-intercepts, vertical and horizontal asymptotes, symmetry of graph.
3. Find critical points and intervals where increasing and decreasing. (1st Derivative)
4. Determine local max. and min. points.
5. Determine concavity and find points of inflection.
6. Sketch the curve. (2nd Derivative)

2

EX. #1: $f(x) = x^3 - 6x^2 + 9x - 2$ Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

y-int: $0^3 - 6(0)^2 + 9(0) - 2$

$= -2$

$(0, -2)$

$f'(x) = 3x^2 - 12x + 9$

$0 = 3(x^2 - 4x + 3)$

$0 = (x-3)(x-1)$

$x = 3 \quad x = 1$

Possible Max/Min

Test # | 0 | 2 | 4

Critical # | 1 | 3

Sign $f'(x)$ | + | − | +

Max at 1, Min at 3

$f''(x) = 6x - 12$

$0 = 6x - 12$

$0 = 6(x-2)$

$x = 2$

Possible Point of Inflection

Test # | 0 | 4

Critical # | 2

Sign $f''(x)$ | − | +

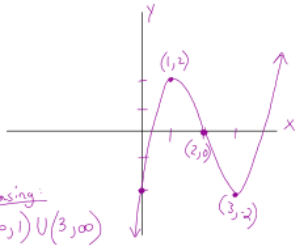
Point of Inflection

Points: $x^3 - 6x^2 + 9x - 2$

Max @ $x=1 \rightarrow 1^3 - 6(1)^2 + 9(1) - 2 = 2$ Max $(1, 2)$

Min @ $x=3 \rightarrow 3^3 - 6(3)^2 + 9(3) - 2 = -2$ Min $(3, -2)$

PoI @ $x=2 \rightarrow 2^3 - 6(2)^2 + 9(2) - 2 = 0$ PoI $(2, 0)$



Increasing: $(-\infty, 1) \cup (3, \infty)$

Decreasing: $(1, 3)$

Concave Up: $(2, \infty)$

Concave Down: $(-\infty, 2)$

3

1. $y = x - 4\sqrt{x}$

$x \geq 0$

$0 = x - 4\sqrt{x}$

$4\sqrt{x} = x$

$16x = x^2$ $\quad x=0$ $\quad x=16$

$0 = x^2 - 16x$

$x(x-16)$

y-int $= 0 - 4\sqrt{0}$

$= 0$

$(0,0)$

x-int $(16,0)$

$y' = 1 - \frac{2}{\sqrt{x}}$

$y' = 0$

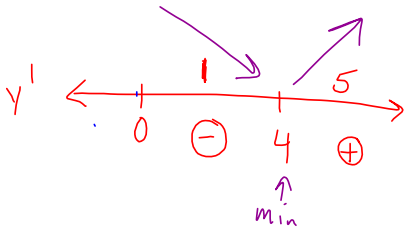
$0 = 1 - \frac{2}{\sqrt{x}}$

$\frac{2}{\sqrt{x}} = 1$

$\sqrt{x} = 2$

$x = 4$

y' is undefined when $x = 0$



$y = 4 - 4\sqrt{4}$

$y = 4 - 8$

$y = -4$

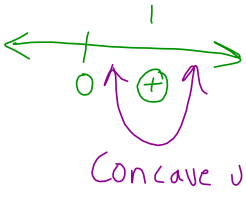
$(4, -4)$ min

$y'' = \frac{1}{x^{3/2}}$

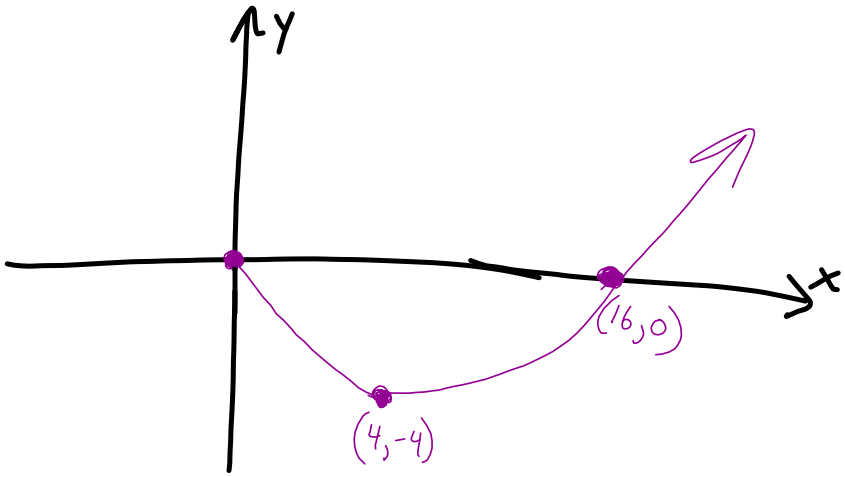
$y'' = 0$

Never

y'' is undefined @ $x = 0$



Concave up

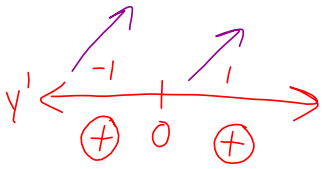


2. $y = \frac{|x-3}{|x}$

$x \neq 0$ V.A. No y-int | H.A. @ $y=1$ (Degrees are same)

x int $0 = x-3$, $x = 3$ $(3,0)$

$y' = \frac{x(1) - (x-3)(1)}{x^2} = \frac{3}{x^2}$



$y' = 0$ Never

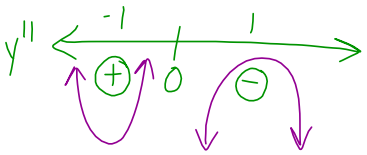
$y' =$ undefined @ $x = 0$

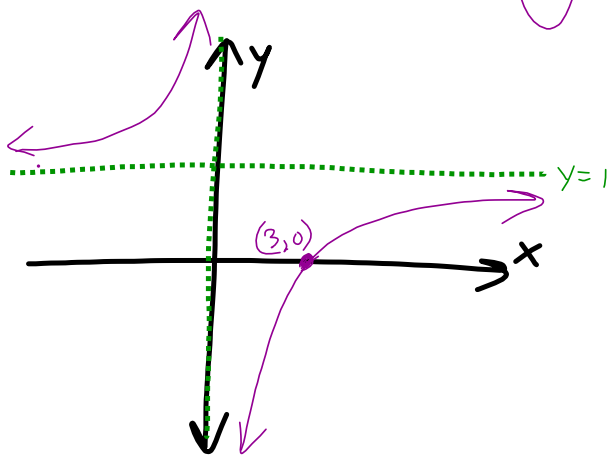
No relative max/min

$y'' = \frac{(x^2)(0) - 3(2x)}{(x^2)^2} = \frac{-6x}{x^4} = \frac{-6}{x^3}$

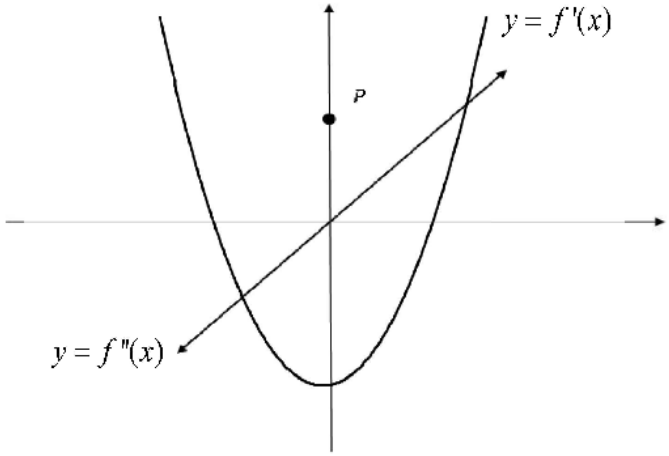
$y'' = 0$ Never

$y'' =$ undefined @ $x = 0$





Graphing f from graphs of f' and f''



Can you sketch a possible graph of f that passes through the point P?

Test #

Critical # _____

Sign $f'(x)$

Test #

Critical # _____

Sign $f''(x)$

Interval/Point				
$f'(x)$				
$f''(x)$				
Inc/Dec				
Concave up Concave down				